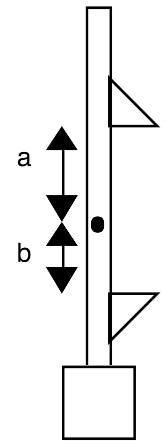
The Kater Pendulum

Background



People have been using pendulums as an easy and accurate way to measure the acceleration due to gravity for hundreds of years. Recall that the formula for the period of a physical pendulum is $T = 2\pi\sqrt{\frac{I}{rmg}}$. While it is easy to accurately measure the period of a pendulum, it was difficult to very accurately measure the location of the center of mass of the pendulum, and to very accurately calculate the moment of inertia.

Around 1817, Henry Kater used a reversable pendulum (now called a Kater's Pendulum) to make extremely accurate measurements of the acceleration due to gravity (to simplify a little.) The Kater Pendulum is a metal rod with two different knife edges and adjustable masses on the ends. It turns out that if the masses are adjusted so that the pendulum has the same period for both knife edges, then the period of oscillation depends only on the distance between the knife edges, and not the location of the center of mass. $T = 2\pi\sqrt{\frac{a+b}{g}}$

Derivation

Calling the periods of oscillation about each knife edge T_a and T_b, we have

$$T_a = 2\pi\sqrt{\frac{I_{cm} + ma^2}{amg}} \quad \text{and} \quad T_b = 2\pi\sqrt{\frac{I_{cm} + mb^2}{bmg}}$$

where I_{cm} is the moment of inertia of the pendulum about its center of mass, m is the mass of the pendulum, and a and b are the distances from each knife edge to the center of mass. Since the pendulum will be adjusted so that the two periods are the same, we can say that

$$2\pi\sqrt{\frac{I_{cm} + ma^2}{amg}} = 2\pi\sqrt{\frac{I_{cm} + mb^2}{bmg}}$$

By letting $k = I_{cm}/m$, we can say

$$2\pi\sqrt{\frac{k + a^2}{ag}} = 2\pi\sqrt{\frac{k + b^2}{bg}}$$

Now simplify and solve for k:

$$\frac{k + a^2}{a} = \frac{k + b^2}{b}$$

$$kb + a^2b = ka + b^2a$$

$$k(b - a) = b^2a - a^2b = ab(b - a)$$

$$k = ab$$

Now take this expression for k and substitute back into one of the original equations:

$$T = 2\pi\sqrt{\frac{ab + a^2}{ag}} = 2\pi\sqrt{\frac{b + a}{g}}$$

So the period only depends on the distance between the knife edges and the acceleration due to gravity!